Understanding the Slutsky Decomposition: Substitution & Income Effect

Placement of the “Final Bundle” when \( p_x \uparrow \): Substitute or Complement Goods?

- When \( p_x \uparrow \), BC rotates inwards – becomes steeper
- Let the original bundle be \( P(x_p, y_p) \)
- The new budget constraint is divided into three regions, Region A, B and C
- These region are developed based on the location of the original bundle, bundle \( P \).
- The relation between the two goods, whether \( x \) and \( y \) are substitutes or complements or \( x \) is a giffen good depends on which region the final bundle falls on.

So to find the relationship between the two goods, we compare the final bundle to the original bundle.

If the final bundle falls in :

**Region A**
- Say, at Bundle Q where:
  - \( x_Q < x_P \)
  - \( y_Q > y_P \)
- We see that as price of good \( x \) increases, the individual buys less \( x \) and more \( y \)
- i.e. he/she switches from a more expensive \( x \) to a cheaper \( y \)
- so \( x \) and \( y \) are **SUBSTITUTE** goods.
- Since quantity demanded of \( x \) ↓ as \( p_x \uparrow \), \( x \) is an **ORTHODOX**

**Region B**
- Say, at Bundle R where:
  - \( x_R < x_P \)
  - \( y_R < y_P \)
- We see that as price of good \( x \) increases, the individual buys less of both \( x \) and \( y \)
- i.e. the goods are jointly consumed - **COMPLEMENTS**
- Also, since quantity demanded of \( x \) ↓ as \( p_x \uparrow \), \( x \) is an **ORTHODOX** good

**Region C**
- Say, at Bundle Q where:
  - \( x_S > x_P \)
  - \( y_S < y_P \)
- Since quantity demanded of \( x \) ↑ as \( p_x \uparrow \), \( x \) is an **GIFFEN** good
Placement of the “Intermediate Bundle” when $p_x \uparrow$: Normal or Inferior Goods?

- To distinguish between the substitution and income effect, we draw the intermediate budget constraint, $BC_2$.
- $BC_2$ must have the same slope as the final Budget Constraint, $BC_3$ – thus incorporating the new price ratio.
- However, $BC_2$ must go through the original bundle, $P$ – this denotes that some income is “given” to the individual so that he can maintain the same purchasing power as before.
- Let the final bundle be $Q(x_Q, y_Q)$.
- $BC_2$ is divided into three regions based on the location of the final bundle and the original bundle.
- Where the intermediate bundle lies, in comparison to the final bundle, indicates whether the goods are normal or inferior.

If the intermediate bundle falls in:

**Region A**
- Say, at Bundle M where:
  - $x_M < x_Q$  
  - $y_M > y_Q$
- We see that as income is “given” to the individual $x$ falls and $y$ rises
- i.e., as $I \uparrow$, $x \downarrow$ - so $x$ is an **INFERIOR** good
- as $I \uparrow$, $y \uparrow$ - so $y$ is a **NORMAL** good

**Region B**
- Say, at Bundle N where:
  - $x_N > x_Q$
  - $y_N > y_Q$
- We see that as income is “given” to the individual both $x$ and $y$ rise.
- i.e., as $I \uparrow$, $x \uparrow$ - so $x$ is a **NORMAL** good
- as $I \uparrow$, $y \uparrow$ - so $y$ is a **NORMAL** good

**Region C**
- Say, at Bundle L where:
  - $x_L > x_Q$
  - $y_L < y_Q$
- We see that as income is “given” to the individual $x$ rises but $y$ falls.
- i.e., as $I \uparrow$, $x \uparrow$ - so $x$ is a **NORMAL** good
- as $I \uparrow$, $y \downarrow$ - so $y$ is an **INFERIOR** good

So, to find out whether the good is normal or inferior we compare the intermediate bundle to the final bundle. (Why?)
Drawing the IC and BC to show the Slutsky Decomposition: Tips

1. First, draw the original BC and final BC and decide the location of the original and final bundle – based on the relationship between the two good
   - When the relationship between the two goods is not mentioned (choose the relationship which will make it easiest for you to draw in the ICs later on)
2. Then draw the intermediate BC – and draw in the intermediate bundle based on whether good x is normal, inferior or a giffen good.
3. Lastly draw in the ICs – make sure they are always tangent to the BC at the original, intermediate and final bundle.
   - Also, make sure your ICs do not intersect.

So, what is the SLUTSKY DECOMPOSITION?
Since Slutsky was the first economist to figure out that the total effect of a price change is caused by two separate effects: the substitution effect (SE) and the income effect (IE) – the process of breaking the total effect (TE) down into the SE and the IE is referred to as the Slutsky Decomposition.

What will be the sign of the SE and IE?

**Sign of SE**
Sign of the SE is always negatives

Why – let’s see...

- When \( p_x \uparrow, Q_d \) of \( x \) \downarrow because compared to good \( y \), good \( x \) has become less attractive
- When \( p_x \downarrow, Q_d \) of \( x \) \uparrow because compared to good \( y \), good \( x \) has become more attractive
- Since there is always this inverse relationship between \( p_x \) and \( Q_d \) of \( x \), the sign of the SE is \(-ve\).

**Sign of IE**
Sign of the IE may be positive or negative – depends on whether the good is normal or inferior:

- If good \( x \) is normal, i.e., when \( I \uparrow \), \( x \uparrow \) - income and demand move in the same direction – i.e. IE is \(+ve\).
- If good \( x \) is inferior, i.e., when \( I \uparrow \), \( x \downarrow \) - income and demand move in opposite directions – i.e. IE is \(-ve\).

Which effect is stronger?

**Normal Good**
- SE and IE reinforce each other – both work in the same direction

**Inferior Good**
- SE and IE work in opposite directions

**Giffen Good**
- IE outweighs SE – this explain how \( Q_d \) of \( x \) \uparrow as \( p_x \) \uparrow,
Slutsky Decomposition for a Normal Good when Price Increases

Since $x$ is a *Normal* good, as income is “given” to individual, he should buy more of $x$ at $R$ than at $Q$.

Location of $Q$ indicates that as $p_x \uparrow$, individual buys less $x$ and more $y$ than at $P$—i.e. the goods are *Substitutes*.

Note that the *IE* reinforces the *SE*—both contribute to the reduction in consumption of $x$ when $p_x \uparrow$.

*What about $y$?*

Since at $R$, when income is “given” to the person, more $y$ is bought than at $Q$—$y$ is a *Normal* good as well.
Slutsky Decomposition for an Inferior Good when Price Increases

Drawing Tip:
Note that to have more space to draw my ICs, I made my initial BC flatter than before and place P more to the right.

The Red Dotted lines help to determine where the intermediate bundle should be.

Since x is an Inferior good, as income is “given” to individual, he should buy less of x than at Q.

Location of Q indicates that as \( p_x \uparrow \), individual buys less x and more y than at P—i.e. the goods are Substitutes.

What about y?
Since at R, when income is “given” to the person, more y is bought than at Q – y is a Normal good.

Note that the IE partially offsets the SE –
- when \( p_x \uparrow \), less x bought since x less attractive than y – thus the SE reduces consumption of x
- when \( p_x \uparrow \), real income falls, and so more of x is bought – thus the IE increases amount of x bought.
Slutsky Decomposition for a Giffen Good when Price Increases

The Red Dotted lines help to determine where the intermediate bundle should be

Since $x$ is an *Inferior* good, as income is “given” to individual, he should buy less of $x$ at $R$ than at $Q$

**Location of Q** indicates that as $p_x \uparrow$, individual buys more $x$ than at $P$—i.e. $x$ is a *Giffen good*.

Note that the $IE$ outweighs the $SE$ –
- when $p_x \uparrow$, less $x$ bought since $x$ less attractive than $y$ – thus the $SE$ reduces consumption of $x$
- when $p_x \uparrow$, real income falls, and so more of $x$ is bought – thus the $IE$ increases amount of $x$ bought
- Since the $IE$ is stronger than the $SE$, $Q_d$ of $x \uparrow$ as $p_x \uparrow$.

*What about $y$?*

Since at $R$, when income is “given” to the person, more $y$ is bought than at $Q$ — $y$ is a *Normal* good
Slutsky Decomposition for Normal Good when Price Decreases

When \( p_x \downarrow \), BC rotates outwards from \( BC_1 \) to \( BC_3 \) - to distinguish between the SE and IE – we draw the intermediate BC at \( BC_2 \) -the BC that passes through the original bundle, Bundle P in order to retain the same purchasing power as before, but has the slope of \( BC_3 \) in order to incorporate the new price ratio.

The Red Dotted lines help to determine where the intermediate bundle should be

Since \( x \) is a Normal good, as income is “taken away” from the individual, he should buy less of \( x \) at \( R \) than at \( Q \)

Note that the IE reinforces the SE –
- when \( p_x \downarrow \), more \( x \) bought since \( x \) more attractive than \( y \) – thus the SE increases consumption of \( x \)
- when \( p_x \downarrow \), real income rises, and so more of \( x \) is bought – thus the IE increases amount of \( x \) bought
- This explains why the IE and SE work together to increase the quantity demanded of \( x \)

Location of Q indicates that as \( p_x \downarrow \), individual buys more \( x \) and less \( y \) than at \( P \)-- i.e. \( x \) and \( y \) are Substitute goods

What about \( y \)?

Since at \( R \), when income is “taken away” from the person, more \( y \) is bought than at \( Q \) – \( y \) is an Inferior good
Slutsky Decomposition for Inferior Good when Price Decreases

Location of Q indicates that as $p_x \downarrow$, individual buys more $x$ and $y$ than at $P$– i.e. $x$ and $y$ are Complements.

The Red Dotted lines help to determine where the intermediate bundle should be.

Since $x$ is an Inferior good, as income is “taken away” from the individual, he should buy more of $x$ at $R$ than at $Q$.

Note that the $IE$ partially offsets the $SE$ –
- when $p_x \downarrow$, more $x$ bought since $x$ more attractive than $y$ – thus the $SE$ increases consumption of $x$.
- when $p_x \downarrow$, real income rises, and so less of $x$ is bought – thus the $IE$ decreases amount of $x$ bought.
- This explains why the $IE$ and $SE$ work in opposite directions.

What about $y$?

Since at $R$, when income is “taken away” from the person, less $y$ is bought than at $Q$ – $y$ is an Normal good.

Since at $P$, when price of $x$ decreases, more $x$ is bought since $x$ more attractive than $y$ – thus the $SE$ increases consumption of $x$.
Slutsky Decomposition for Giffen Good when Price Decreases

Location of Q indicates that as \( p_x \downarrow \), individual buys less \( x \) than at \( P \) – i.e. \( x \) is a Giffen good.

The Red Dotted lines help to determine where the intermediate bundle should be.

Since \( x \) is an Inferior good, as income is “taken away” from the individual, he should buy more of \( x \) at \( R \) than at \( Q \).

Note that the IE outweighs the SE –
- when \( p_x \downarrow \), more \( x \) bought since \( x \) more attractive than \( y \) – thus the SE increases consumption of \( x \)
- when \( p_x \downarrow \), real income rises, and so less of \( x \) is bought – thus the IE decreases amount of \( x \) bought
- Since the IE is stronger than the SE, consumption of \( x \) falls as \( p_x \downarrow \).

What about \( y \)?
Since at \( R \), when income is “taken away” from the person, less \( y \) is bought than at \( Q \) – \( y \) is an Normal good.