# Introduction to Functions

In this section we focus on understanding:

- Difference between relation and function
- Definition of a function
- Differentiate between function and relation based on :
    - Arrow Chart
    - Vertical Line Test
    - Solving for '$y$'

## Relation vs. Function

To understand relations and functions we first have to become familiar with certain terminology such as 'ordered pairs', 'domain' and 'range'. This is covered below.

### *Relations*

A "relation" is just a relationship between sets of information. Think of all the people in one of your classes, and think of their heights. The pairing of names and heights is a relation.

### *Ordered Pairs*

In relations and functions, the pairs of names and heights are "ordered", which means one comes first and the other comes second. Having an ordered pair means that the 'order' has a meaning – if the order is not maintained, the meaning may change.

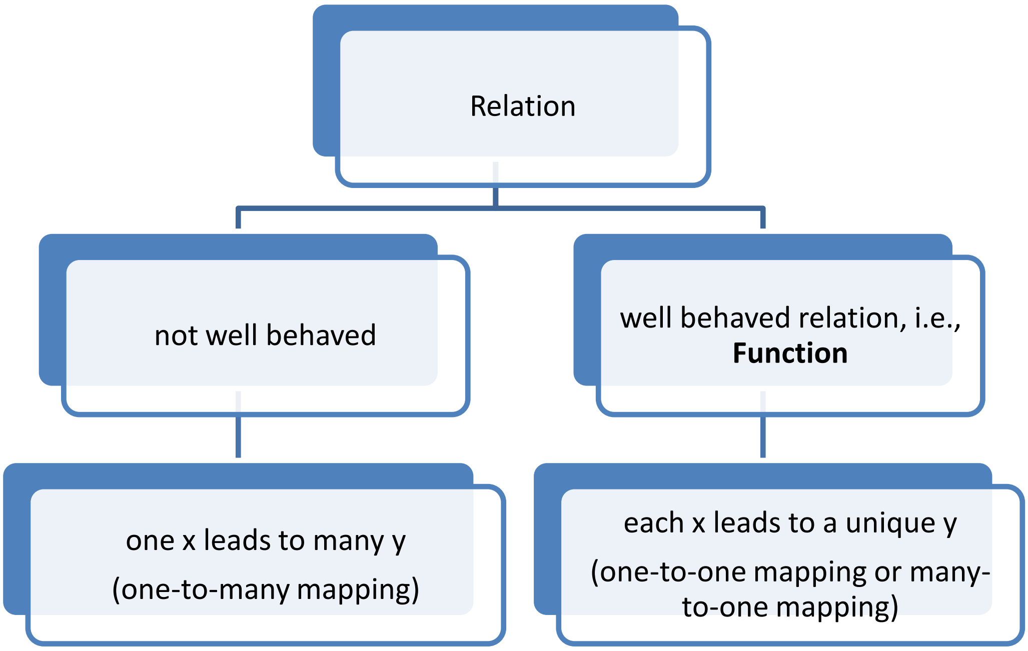
- For example, suppose we are dealing with an ordered pair : $(age, height)$
- Suppose I give the information that Jim is 25 years old and weight is 127 pounds. If I want to present it as an ordered pair, I would write it as: (25,127)
- If I were to 'mess up' the order of the pair and write it as (127,25), the information that I am giving in incorrect. This ordered pair gives the information that Jim is 127 years and weighs 25 pounds which is incorrect and in case of age almost impossible in modern times.
- When we are plotting points on the Cartesian plane, pairs such as (2,1) are also 'ordered pairs'. By looking at (2,1) we know that the x co-ordinate is 2 and the y co-ordinate is 1. If we 'messed up' the order and wrote it as (1,2), the meaning would completely change and we would be plotting an entirely different point on our graph – one where the $x - coordinate$ is 1 and $y - coordinate$ is 2.
- The gist of this discussion is – unlike what we saw in Set Theory, order does matter.
- If you were discussing a 3-Dimentional world, you would be dealing with 'ordered triples'.

### *Domains and Ranges*

The set of all starting points is called the 'domain' and the set of all ending points is called the 'range'. The domain is what you start with, the range is what you end with. The domain is the $x's$, the range is the $y's$. (I'll explain more on determining domains and ranges in the next section)

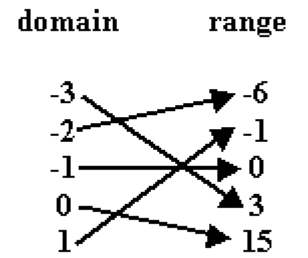
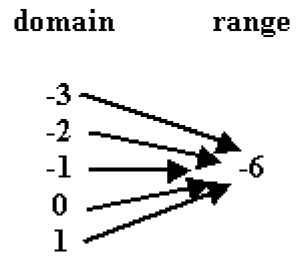
### *Functions*

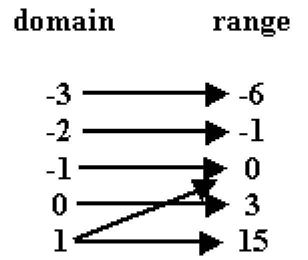
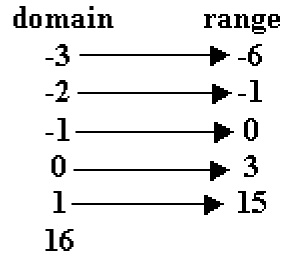
A function is a "well-behaved" relation. This means that while all functions are relation, not all relations are functions. Functions are a sub-classification of relations. This can be best explained by the following tree diagram.




When we say the function is a 'well-behaved relation', we mean that given a starting point, we know exactly where to go; given an *x*, we get only and exactly one *y*.

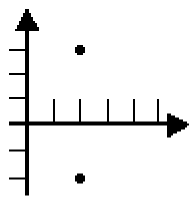
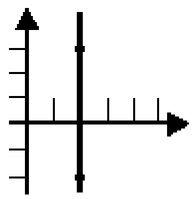
## Is it a Function? – The Arrow Chart

| | |
|---|---|
|   | This is a function. You can tell by tracing from each *x* to each *y*. There is only one *y* for each *x*; there is only one arrow coming from each *x*. |
|   | Ha! Bet I fooled some of you on this one! This *is* a function! There is only one arrow coming from each *x*; there is only one *y* for each *x*. It just so happens that it's always the same *y* for each *x*, but it is only that one *y*. So this is a function; it's just an extremely *boring* function!<br>(This is a Constant Function) |

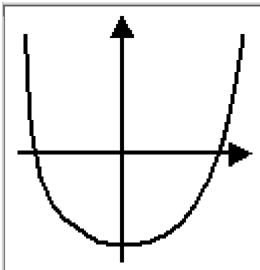
| | |
|---|---|
|  | This one is not a function: there are *two* arrows coming from the number 1; the number 1 is associated with two *different* range elements. So this is a relation, but it is not a function. |
|  | Okay, this one's a trick question. Each element of the domain that has a pair in the range is nicely well-behaved. But what about that 16? It *is* in the domain, but it has no range element that corresponds to it! This won't work! So then this is not a function. In fact, it is not even a relation! |

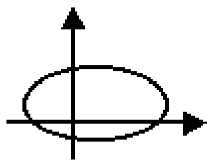
## Is it a Function? – The Vertical Line Test

Looking at this function stuff graphically, what if we had the relation that consists of a set containing just two points: {(2, 3), (2, –2)}? We already know that this is not a function, since $x$ = 2 goes to each of $y$ = 3 and $y$ = –2.

| | |
|---|---|
| If we graph this relation, it looks like: |  |
| Notice that you can draw a vertical line through the two points, like this: |  |

This characteristic of non-function was codified in "The Vertical Line Test": Given the graph of a relation, if you can draw a vertical line that crosses the graph in more than one place, then the relation is not a function. Here are a couple examples:

| | |
|---|---|
|  | This graph shows a function, because there is no vertical line that will cross this graph twice.<br>This means that one $x$-value yields only one unique $y$-value. |

| | |
|---|---|
|  | This graph does not show a function, because any number of vertical lines will intersect this oval twice. For instance, the $y$-axis intersects (crosses) the line twice. This implies that one $x$-value yields two different values of $y$. |

## Is it a Function? – Solving for '$y$'

Even without the vertical line test, we can determine whether a relation is a function or not. For instance, let's take the case of the following relation:

$$2y + 3x = 6$$

$$or, 2y = -3x + 6$$

$$\therefore y = (-{}^{3}/_{2})x + 3$$

Since we get a unique solution for $y$, $2y + 3x = 6$ is indeed a function. If we plug in one value of $x$, we get only one value of $y$!

On the other hand, $y^2 + 3x = 6$ is not a function, because you cannot solve for a unique $y$:

$$y^2 + 3x = 6$$

$$or, y^2 = -3x + 6$$

$$\therefore y = \pm\sqrt{-3x + 6}$$

Here the relation is NOT a function. This is because, for each value of x, we get two different values of y – a positive and negative value. So, if we take $x = -1$, then we get $y = \pm 3$, i.e., one value of x yields two different values of $y$.

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**THE END**